Theory of the Modified Two-Stream Instability in a Magnetoplasmadynamic Thruster

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It is shown that for plasma parameters characteristic of those found in magnetoplasmadynamic (MPD) thrusters the modified two-stream instability may exist in the plasma. The critical parameter for triggering this instability is the ratio of the crossfield current to the ion saturation current. Once triggered, this instability greatly increases the plasma resistivity to the flow of the current and heats both ions and electrons. The anomalous momentum-exchange frequency and heating rates are calculated for characteristic MPD thruster parameters.

Nomenclature

A	= channel area
B	= magnetic field strength
D_a	= ambipolar diffusion coefficient
E	= electric field
E_l	= energy transfer due to classical elastic collisions
E_i	= ionization energy
e	= elementary charge
$\boldsymbol{e}$	= unit vector
f	= particle velocity distribution function
H	= interelectrode separation
I_0	= Bessel function of zeroth order and imaginary argument
I_1	= Bessel function of first order and imaginary argument
J	= transverse current density
J_0	= Bessel function of zeroth order
K_e	= axial heat conduction coefficient
$\boldsymbol{k}$	= wave vector
k	= inverse of Debye length
L_B	= magnetic scale length
L_U	= velocity scale length
m	= particle mass
n	= particle number density
$\dot{n}_e$	= rate of change of n_e due to ionization and recombination
P	= pressure
T	= temperature in energy units
t	= time
$\bar{U}$	= dimensionless transverse current
U	= axial velocity
U_D	= ion-electron differential velocity
$\boldsymbol{v}$	= velocity vector
v_{th}	= thermal velocity
W_k	= saturation energy
x,y,z	= coordinate directions
$\boldsymbol{x}$	= position vector
Z	= plasma dispersion relation
α	= ionization fraction
β_k	= dimensionless kinetic pressure
Γ_i	= ion current evaluated at Bohm velocity
Δ	= perturbation of following quantity
δ	= dimensionless density
ϵ	= dielectric constant
ϵ_0	= permittivity of free space
θ	= wave propagation angle with respect to magnetic field
$\bar{\theta}$	= propagation angle
κ	= wave number
λ	= Debye length
Λ	= Spitzer-Harm logarithm
μ	= viscosity coefficient
μ_0	= permeability of free space
ν	= collision frequency
$\bar{\nu}$	= dimensionless electron neutral collision rate
ν_d	= anomalous momentum exchange frequency
ν_e	= classical electron momentum exchange frequency
ν_H	= classical heating rate
ν_T	= species heating rate
ξ	= dimensionless phase velocity
ξ_B	= dimensionless ∇B drift
ρ	= mass density
ρ_e	= electron gyroradius
σ	= plasma conductivity
φ	= perturbed potential
χ	= susceptibility
ω	= oscillation frequency
$\bar{\omega}$	= dimensionless oscillation frequency
ω_B	= ∇B drift frequency
ω_{LH}	= lower hybrid frequency
ω_p	= plasma frequency
Ω_e	= electron gyrofrequency

Subscripts

i,e,n	= value of quantity for ion, electron, or neutral species
$\perp, \parallel$	= value of quantity in perpendicular and parallel directions

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I. Introduction

MAGNETOPLASMADYNAMIC (MPD) thrusters are electromagnetic thrusters that work by utilizing the Lorentz force produced by charged particles moving in a magnetic field to accelerate the propulsive fluid. Typically, an electric field is applied transverse to the flow as shown in Fig. 1. This field induces a current which, in general, flows both transverse and parallel to the flow. The current creates a self-consistent magnetic field that interacts with the current to give a force that accelerates the flow. There is another class of MPD thrusters called applied field thrusters where the magnetic field is applied externally. However, we shall not analyze these thrusters in this paper.

The steady-state one- and two-dimensional flow of plasma in an MPD thruster has been analyzed in a number of papers (e.g., Refs. 1 and 2 and references therein). A few papers have

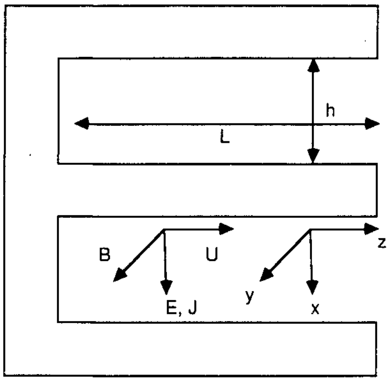

Fig. 1 Coordinate system in a parallel plate MPD thruster.

also considered the possibility that the plasma flow may be unstable due to the growth of current-driven instabilities. This is because the current is a large reservoir of free energy which can be used to drive plasma instabilities. The effects of plasma instabilities on an equilibrium plasma state is well known. For instabilities that involve macroscopic plasma motion, the plasma may kink and twist. For more microscopic plasma motions, there is often a large increase in the plasma resistivity coupled with anomalous energy transfer between the plasma species as well as the growth of electrostatic or electromagnetic oscillations.[3] Two current-driven instabilities that may occur in a plasma are the well-known ion acoustic instability[4] and the Buneman instability.[4] The possibility that these instabilities may be responsible for the observed onset of oscillations near the anode at a critical current was first examined for plasma thrusters in Ref. 5. In more recent theoretical and experimental work,[6,7] it was shown that ion acoustic waves occur in MPD plasma flows and the importance of lower hybrid instabilities is detailed. Another theoretical analysis in Ref. 8 showed that a hydrodynamic acoustic wave instability could be excited in an MPD plasma.

All of the previous work considered acoustic instabilities in the MPD flow. Acoustic instabilities are easiest to excite for current flows along the magnetic field.[9] However, the MPD plasma has a current flowing transverse to the field. This has led us to consider the modified two-stream instability[10] in an MPD plasma flow.

The modified two-stream instability can be understood as the analog of the hydrodynamic two-stream instability in a magnetized fluid.[11] Whereas the two-stream or Buneman instability requires that the differential stream velocity associated with the current exceed the electron thermal velocity in order to be excited, the modified two-stream instability does not. This is because the excited waves propagate nearly perpendicular to the magnetic field. Since the thermal effects of the electrons are limited in their influence to distances of the order of the electron gyroradius, the waves see the electrons as a cold fluid. Hence, the modified two-stream instability can be excited for differential velocities much less than the electron thermal velocity. This makes this instability particularly dangerous since it is so easy to excite and motivates this study of the effect of it on MPD plasma flows.

In Sec. II we develop the underlying equilibrium model and the linear theory of this instability as applied to MPD plasma flows. However, in order not only to know if the instability is excited but also to know the effect of it on the plasma, we develop the nonlinear theory of this instability in Sec. III and obtain expressions for the anomalous momentum exchange and heating associated with this instability. In Sec. IV we apply the theory developed in the last two sections to a one-dimensional model for the MPD plasma flow and show that the flow can be unstable with respect to the modified two-stream instability. Finally, in Sec. V we discuss the consequences of exciting these instabilities in MPD plasmas and suggest experimental measurements that could be used to test this theory.

II. Linear Theory of the Modified Two-Stream Instability

We consider a flowing plasma of ions and electrons as shown in Fig. 1 with a differential velocity between ions and electrons of $U_D = J/(en_e)$. In steady state,

$$E = \frac{J}{\sigma} + UB \tag{1}$$

The current density is related to the magnetic field by

$$J = -\frac{1}{\mu_0}\frac{\partial B}{\partial z} \tag{2}$$

The MPD regime is defined as the regime where the plasma pressure is very small as compared to the kinetic pressure so that the thrust comes from electromagnetic acceleration rather than thermal expansion. Hence, the force in the axial direction gives

$$\frac{\partial}{\partial z}\left(\rho U^2 + \frac{B^2}{2\mu_0}\right) = 0 \tag{3}$$

If we define

$$L_B = -\left(\frac{1}{B}\frac{\partial B}{\partial z}\right)^{-1} \tag{4}$$

$$L_U = \left(\frac{1}{U}\frac{\partial U}{\partial z}\right)^{-1} \tag{5}$$

and

$$\beta_k = \frac{\rho U^2}{B^2/(2\mu_0)} \tag{6}$$

then from Eq. (3) the scale lengths are related by

$$\frac{1}{L_B} = \frac{\beta_k}{2}\frac{1}{L_U} \tag{7}$$

In an MPD thruster, we expect that $\beta_k = O(1)$, which implies that there must be a strong magnetic field gradient even though the thermal pressure in the thruster may be very low. This strong magnetic field gradient leads to a transverse ∇B drift of the electrons,[12] which must be included in the stability analysis.

We shall choose the steady-state equilibrium to consist of a flowing plasma with all variations in the z direction. The equilibrium variation in density and temperature will be ignored. This means that we shall ignore the density and temperature gradient drifts across the plasma and the associated frequencies. For the ions this is justified since the length of the thruster will be taken as such that the ions will not have time to complete their gyrorbits before they leave the thruster. However, the electrons will complete their gyrorbits and will be taken as magnetized. Hence, the neglect of the gradient drifts will only be valid in the limit that the frequency of the instability that we are considering is large as compared to the gradient drift frequencies. Since we anticipate that the frequency of the instability is of the order of the lower hybrid frequency, this is true over most of the plasma with the possible exception of the end of the thruster. However, we choose to consider the ∇B drift since it gives rise to a real particle drift and so will affect the electron resonance with the waves. By contrast, the density gradient drift is a fluid drift and so will not affect the particles directly. The low plasma pressure means that we can assume that electrostatic and electromagnetic waves in the plasma will be decoupled.[11] Hence, we look for electrostatic perturbations to the equilibrium of the form

$$\varphi(x,t) = \varphi(z,k,\omega)\exp\left[i(k\cdot x - \omega t)\right]$$

The wave vector $\boldsymbol{k}$ has the form

$$\boldsymbol{k} = k\cos(\theta)\boldsymbol{e}_x + k\sin(\theta)\boldsymbol{e}_y$$

We can also write this as

$$\boldsymbol{k} = k_\perp \boldsymbol{e}_x + k_\parallel \boldsymbol{e}_y$$

If we choose the electron and ion distribution functions to be local Maxwellians, then the perturbed Poisson equation will give the dispersion relation with the one additional assumption that the wavelength of the waves in the axial direction is short compared to the thruster length. This is the well-known local approximation.[12]

The electrostatic dispersion relation is easy to obtain[13] by integration over the unperturbed particle orbits and is given by

$$\epsilon = 1 + \chi_e + \chi_i = 0 \tag{8}$$

where the susceptibilities are

$$\chi_e = \frac{k_e^2}{k^2}\left[\frac{1 + (\omega + i\nu_{en})/(k_\parallel v_{\mathrm{th}_e})L_e}{1 + (i\nu_{en})/(k_\parallel v_{\mathrm{th}_e})L_e}\right] \tag{9}$$

and

$$\chi_i = \frac{k_i^2}{k^2}\left[1 + \frac{\omega - k\cos(\theta)U_D}{kv_{\mathrm{th}_i}} Z\left(\frac{\omega - k\cos(\theta)U_D}{kv_{\mathrm{th}_i}}\right)\right] \tag{10}$$

The function L_e is defined as

$$L_e = \int_0^\infty \mathrm{d}\xi e^{-\xi} J_0^2(\sqrt{2b_e\xi})\, Z\left(\frac{\omega + i\nu_{en} + \omega_B\xi}{k_\parallel v_{\mathrm{th}_e}}\right) \tag{11}$$

where $b_e = k_\perp^2\rho_e^2/2$, $Z(x)$ is the plasma dispersion relation[14] and

$$\omega_B = -k_\perp \frac{T_e}{eB}\frac{1}{L_B} \tag{12}$$

In deriving the susceptibilities, we work in a frame of reference fixed to the electrons and take the ions as unmagnetized and the electrons as magnetized and subject to electron neutral collisions as well as a ∇B drift. Both species are taken to be warm. The model for the collision operator is the density-conserving Bhatnager, Gross, Krook[12] model, which is a reasonable mode for electron-neutral collisions.[15] We have chosen to neglect ion-neutral collisions. This is a reasonable assumption as long as the ions and neutrals are tightly coupled so that there is no ion slip. In these circumstances, the ion-neutral collision rate will be much smaller than the electron-neutral collision rate, which for typical MPD parameters is of the same order as the lower hybrid frequency. Hence, the effect of ion-neutral collisions is expected to be negligible on the instability time scale, which is the lower hybrid frequency.

The modified two-stream instability has a frequency on the order of the lower hybrid frequency, which is defined as

$$\omega_{\mathrm{LH}} = \omega_{p_i}/(1 + \omega_{p_e}^2/\Omega_e^2) \tag{13}$$

We assume (and can verify a posteriori for one-dimensional MPD flows) that the ratio $\omega_B/\omega_{\mathrm{LH}}$ is small. With this assumption we can approximate the function L_e by

$$L_e = e^{-b_e} I_0(b_e)\left[Z\left(\frac{\omega_e}{k_\parallel v_{\mathrm{th}_e}}\right) + \frac{\omega_B}{k_\parallel v_{\mathrm{th}_e}}\frac{\mathrm{d}}{\mathrm{d}x} Z\left(\frac{\omega_e}{k_\parallel v_{\mathrm{th}_e}}\right) g(b_e)\right] \tag{14}$$

where $\omega_e = \omega + i\nu_{en}$ and

$$g(b_e) = 1 - b_e\left[1 - \frac{I_1(b_e)}{I_0(b_e)}\right] \tag{15}$$

We follow Ref. 10 and expand in the smallness of the angle θ. This implies that the wave will propagate almost perpendicular to the magnetic field. This is because, as shown in Ref. 10, at small angles with respect to the magnetic field, the wave is heavily damped by electron Landau damping. With this expansion we introduce the following parameters:

$$\kappa = \frac{k\rho_e}{\sqrt{2}}\frac{\Omega_e}{\omega_{p_e}}\left(1 + \frac{\omega_{p_e}^2}{\Omega_e^2}\right)^{1/2} \tag{16}$$

$$\bar{\omega} = \omega/\omega_{\mathrm{LH}} \tag{17}$$

$$\bar{\theta} = \theta\left(\frac{m_i}{m_e}\right)^{1/2} \tag{18}$$

$$\xi_B = \frac{\rho_e}{2L_B}\left(\frac{m_i}{m_e}\right)^{1/2} \tag{19}$$

$$\bar{\nu} = \frac{\nu_{en}}{\omega_{\mathrm{LH}}} \tag{20}$$

A very important parameter is $\bar{U}$ given by

$$\bar{U} = \frac{U_D}{\sqrt{2}v_{\mathrm{th}_i}} \tag{21}$$

The dimensionless current density is also given by

$$\bar{U} = J/(2/\sqrt{\frac{T_e}{T_i}}\Gamma_i)$$

The dispersion relation then becomes

$$\begin{aligned} 0 = 1 &+ \frac{1 - \Gamma_0[\kappa^2(\delta^2/1 + \delta^2)]}{\kappa^2}\frac{1 + \delta^2}{\Upsilon} - \frac{T_e}{T_i}\frac{1 + \delta^2}{2\kappa^2}\frac{\mathrm{d}}{\mathrm{d}x}Z(\xi_i) \\ &- \Gamma_0\left(\kappa^2\frac{\delta^2}{1 + \delta^2}\right)\frac{1 + \delta^2}{2\kappa^2}\frac{\mathrm{d}}{\mathrm{d}x}Z(\xi_e) \\ &\times \frac{1 - 2\xi_B\xi_e/\bar{\theta}g[\kappa^2(\delta^2/1 + \delta^2)]}{\Upsilon} \end{aligned} \tag{22}$$

where

$$\delta = \omega_{p_e}/\Omega_e \tag{23}$$

$$\xi_i = \frac{\bar{\omega}}{\kappa}\frac{1}{\sqrt{2}}\left(\frac{T_e}{T_i}\right)^{1/2} - \frac{\bar{U}}{\sqrt{2}} \tag{24}$$

$$\xi_e = \frac{\bar{\omega}}{\kappa}\frac{1}{\sqrt{2}}\frac{1}{\bar{\theta}} + \frac{i\bar{\nu}}{\kappa}\frac{1}{\sqrt{2}}\frac{1}{\bar{\theta}} \tag{25}$$

$$\begin{aligned} \Upsilon = 1 &+ \frac{\bar{\omega}}{\kappa}\frac{1}{\sqrt{2}}\frac{1}{\bar{\theta}}\Gamma_0\left(\kappa^2\frac{\delta^2}{1 + \delta^2}\right) \\ &\times\left[(Z(\xi_e) + \frac{\xi_B}{\bar{\theta}}\frac{\mathrm{d}}{\mathrm{d}x}Z(\xi_e)g\left(\kappa^2\frac{\delta^2}{1 + \delta^2}\right)\right] \end{aligned} \tag{26}$$

The function $\Gamma_0(b_e)$ is $e^{-b_e}I_0(b_e)$. From the dispersion relation, we see that the complex frequency is a function of κ and $\bar{\theta}$. It is parametrically a function of the underlying equilibrium, which is characterized by δ, the temperature ratio T_e/T_i, $\bar{U}$, $\bar{\nu}$, and ξ_B. This dispersion relation is an extension of the one in Ref. 10 to include electron-neutral collisions and (weak) ∇B drifts.

The modified two-stream instability is a nonresonant instability, which is obtained in the limit $|\xi_i| \gg 1$ and $|\xi_e| \gg 1$. If $|\xi_i| \leq 1$, then the wave is damped by ion Landau damping whereas if $|\xi_e| \leq 1$, then the wave is damped by electron Landau damping. From Eq. (25) we see that this imposes a condition on the propagation angle [basically that $\theta = O(1)$]. Similarly from Eq. (24) we see that a condition is imposed on $\bar{U}$. In the limit $\bar{\nu} = 0$, $\xi_B = 0$, and $\theta = 1$ and with the large argument expansions of the plasma dispersion function,[14] the dispersion relation reduces to

$$1 - \frac{T_e}{T_i}\frac{1}{[\bar{\omega}(T_e/T_i)^{1/2} - \kappa\bar{U}]^2} - \frac{1}{\bar{\omega}^2} = 0$$

which has the solution at maximum growth

$$\bar{\omega} = \frac{\sqrt{3}}{2} + \frac{i}{2} \tag{27}$$

with the wave number at maximum growth being

$$\kappa = \frac{\sqrt{3}}{\bar{U}}\left(\frac{T_e}{T_i}\right)^{1/2} \tag{28}$$

From Eq. (27) we see that the modified two-stream instability has an oscillation frequency of the order of the lower hybrid frequency and a maximum growth rate of the same order. This is characteristic of a hydrodynamic instability that the growth rate can be of the same order as the oscillation frequency and indicates why these instabilities are dangerous. We also note that the growth [Eq. (28)] does depend on this ratio. This is in marked contrast to the ion acoustic instability which has previously been considered in MPD thrusters. The ion acoustic instability is very strongly dependent on the electron to ion temperature ratio and is damped when the ratio is too small.

In Fig. 2 we examine the effect on the growth rate maximized over κ and $\bar{\theta}$ of varying the dimensionless current $\bar{U}$ and the temperature ratio. We choose $\delta = 60$, $\xi_B = 3 \times 10^{-2}$, and $\bar{\nu} = 1.08$. We note that in dimensional parameters of $B = 0.1$ T, $T_i = 0.3$ eV and for $T_e/T_i = 1$, $\bar{U} = 10$. We have $n_e = 3.5 \times 10^{20}$ m^{-3} and $J = 475$ kA/m^2 (for an anode of 0.2×0.2 m^2, this is a current of 19 kA). We see that for $U_D/v_{th_i} \geq 3$, the wave is unstable over the range of temperature ratios with not much difference between the different temperature ratios. For $T_e/T_i = 10$, the wave is somewhat less unstable due to the slightly broader electron Landau resonance. Hence, we can conclude that the modified two-stream instability is likely to exist in a typical MPD thruster as long as the transverse current is sufficiently large.

III. Nonlinear Theory of the Modified Two-Stream Instability

In Sec. II we showed that the modified two-stream instability may be unstable for typical MPD parameters. Once the linear stability boundary is crossed, the instability will start to grow, saturate at some level, and cause increased particle and energy scattering. Therefore it is important to obtain the nonlinear state of the instability in order to assess its impact on the plasma flow. This nonlinear state will be reached as long as the residence time of the plasma in the thruster is substantially longer than the time scale over which the wave grows.

There are many possible mechanisms by which plasma instabilities can saturate. Three of the most common are saturation due to quasilinear stabilization,[3] saturation due to resonance broadening,[16] and saturation due to electrostatic trapping.[10] In this paper we shall consider the latter since our perpendicularly propagating waves are similar to lower hybrid waves which saturate by electrostatic trapping. Trapping of the ions in the potential structure of the wave will start to occur when

$$e\varphi \simeq \frac{1}{4}m_i\left(\frac{\omega}{k_\perp} - U_D\right)^2 \tag{29}$$

The electrostatic energy at saturation is

$$\frac{\langle W_k\rangle}{n_eT_e} = \frac{1}{64}\left\{\frac{\kappa^2}{(T_e/T_i)(1+\delta^2)}[\bar{U} - \bar{\omega}_{real}/\kappa(T_e/T_i)^{1/2}]^4\right\}_{\kappa_{max}} \tag{30}$$

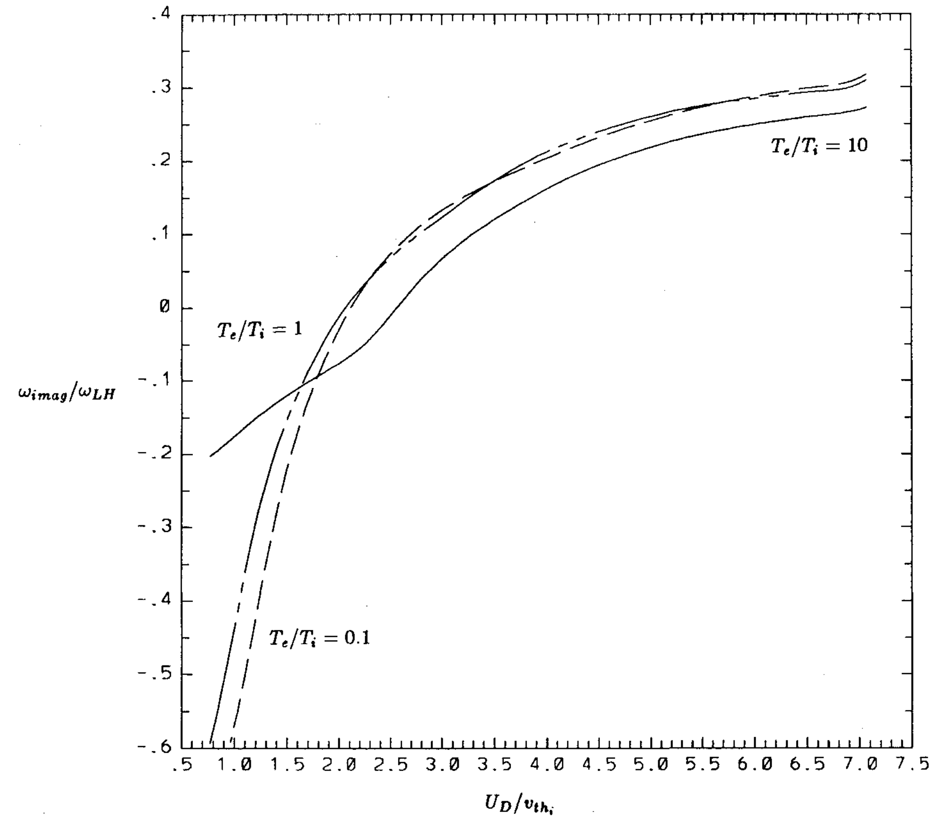

Fig. 2 The ω_{imag} against U_D/v_{th_i} for $T_e/T_i = 0.1$, 1, 10, and $\bar{\nu} = 1.08$, $\xi_B = 3 \times 10^{-2}$.

where $\langle\ \rangle$ denotes an ensemble average and the expression is to be evaluated at the κ, which gives maximum growth. For typical values of the parameters, the energy at saturation is $\langle W_k\rangle/n_eT_e = 1 \times 10^{-5}$. This is typically what is found for nonlinear saturation of linear microinstabilities.[4]

In response to the unstable field fluctuations, the distribution functions evolve according to

$$\left[\frac{\partial}{\partial t} + v\cdot\frac{\partial}{\partial x} - \frac{e}{m_e}\left(\langle E\rangle + \frac{v\times\langle B\rangle}{c}\right)\cdot\frac{\partial}{\partial v}\right]f = \left(\frac{\partial f}{\partial t}\right)_{\text{an}} \tag{31}$$

where

$$\left(\frac{\partial f}{\partial t}\right)_{\text{an}} = \frac{e}{m}\left\langle \Delta E\cdot\frac{\partial \Delta f}{\partial v}\right\rangle \tag{32}$$

Integration of this expression[17] over velocity space gives the scattering frequencies due to turbulent scattering. If we define ν_D by

$$\frac{\partial}{\partial t}\, nm_e v_{y_e} = \nu_D nm_e U_D \tag{33}$$

then we obtain

$$\frac{\nu_D}{\omega_{\text{LH}}} = \frac{\langle W_k\rangle}{n_eT_e}\frac{\kappa}{\bar U}\, 2\,\text{imag}(\chi_i)\left(\frac{T_e}{T_i}\right)^{1/2}\frac{m_i}{m_e} \tag{34}$$

where $\kappa\text{imag}(\chi_e)$ is to be evaluated at the κ, which gives the maximum growth. We note that since $\text{imag}(\chi_i) = -\text{imag}\,(\chi_e)$, the rate of change of momentum for the electrons ($\nu_D nm_eU_D$) is equal and opposite to the rate of change of momentum for the ions. Hence, the wave acts to remove momentum from the flowing ions (since we are in the rest frame of the electrons) and deposits it in the electrons. This will be seen therefore as an increased resistivity for the flow of current. The current as seen through the differential velocity is what provides the energy source for the instability, and the instability acts in such a way so as to remove the free energy in the current by making it harder for the current to flow.

In a similar way, we can define the particle heating rates by

$$\frac{\partial}{\partial t} nT_j = \nu_{T_j} nT_j \tag{35}$$

where $j = i$ or e. The ion heating rate is then

$$\frac{\nu_{T_i}}{\omega_{\text{LH}}} = 4\,\frac{\langle W_k\rangle}{n_eT_e}\left(\frac{T_e}{T_i}\right)^{1/2}\text{imag}\left\{\left[\bar\omega\left(\frac{T_e}{T_i}\right)^{1/2} - \kappa\bar U\right]\chi_i\right\} \tag{36}$$

and the electron heating rate is

$$\frac{\nu_{T_e}}{\omega_{\text{LH}}} = 4\,\frac{\langle W_k\rangle}{n_eT_e}\left[-\bar\omega_{\text{imag}} - \text{imag}\,(\bar\omega\chi_i)\frac{T_e}{T_i}\right] \tag{37}$$

In order to be able to compare these anomalous rates to the classical rates in an electron-ion MPD plasma, we define the classical electron momentum exchange frequency ν_e as the electron-ion collision rate for a strongly ionized flow. In MKS units we have

$$\frac{\nu_e}{\omega_{\text{LH}}} = \frac{3.64\times 10^{-6} n_e \ln\Lambda}{\omega_{\text{LH}} T_e^{3/2}} \tag{38}$$

We also defined the effective classical heating rate of the plasma through the expression

$$\frac{1}{n_eT_e}\frac{\partial}{\partial t}\, n_eT_e = \frac{2}{3}\frac{J^2}{\sigma}\frac{1}{nT_e} = \nu_H \tag{39}$$

which gives

$$\frac{\nu_H}{\omega_{\text{LH}}} = \frac{2}{3}\frac{\epsilon_0\Omega_e}{\sigma}\left(\frac{m_i}{m_e}\right)^{1/2}(1+\delta^2)\frac{\bar U^2}{T_e/T_i} \tag{40}$$

In Fig. 3 we examine the anomalous momentum exchange frequency as a function of $\bar U$ for the same parameters as in Fig.

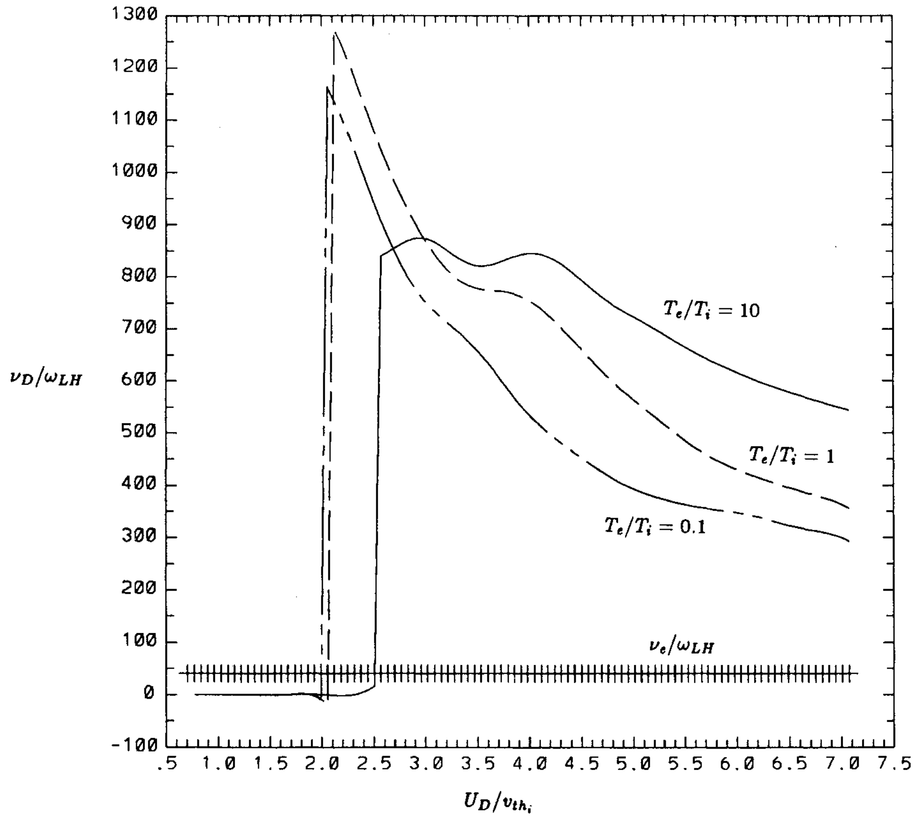

Fig. 3 The ν_D/ω_{LH} and ν_e/ω_{LH} against U_D/v_{th_i} for $T_e/T_i = 0.1$, 1, 10.

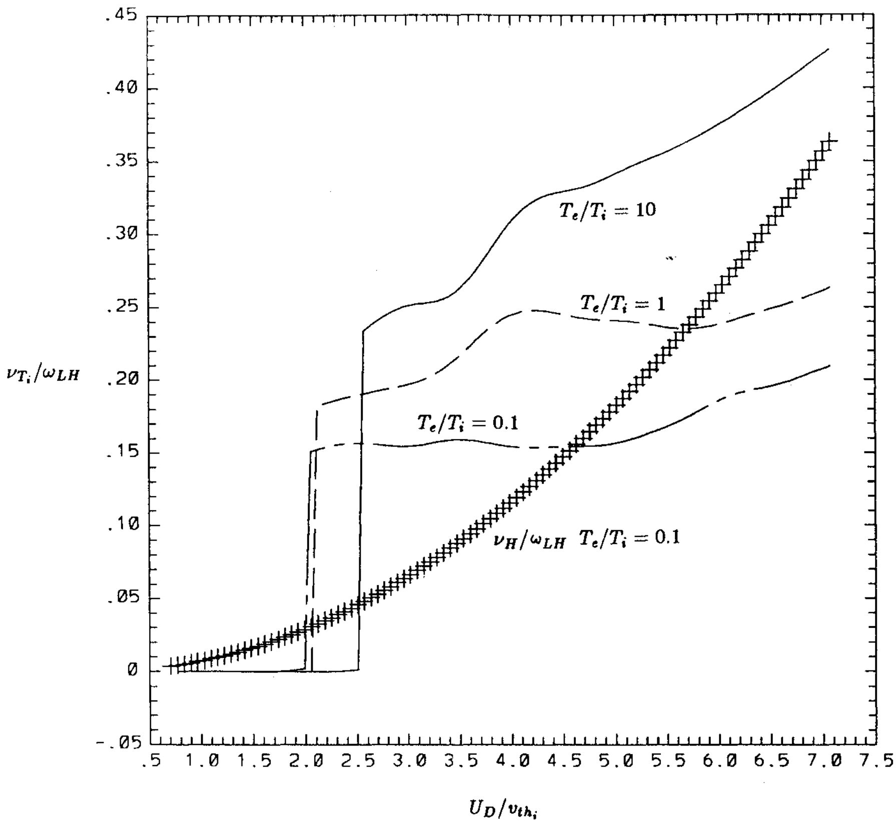

Fig. 4 The ν_{Ti}/ω_{LH} and ν_H/ω_{LH} against U_D/v_{th_i} for $T_e/T_i = 0.1$, 1, 10.

2. The reference frequency is chosen to have the typical value $\nu_e/\omega_{LH} = 40$. This corresponds to a density of $n_e = 6 \times 10^{19}$ m^{-3} for $B = 0.1$ T and $T_e = 1$ eV. We see that once the instability is excited, the anomalous momentum exchange rate is over an order of magnitude larger than the classical value for the range of temperature ratios considered. The other interesting point to note is that the collision frequency is a function of the current (through $\bar{U}$) so that the relation between the current and the electric field will be modified from the simple relation obtained from Eq. (1) if the conductivity is taken as a constant. Such a large increase in the plasma resistivity once the instability is excited will give rise to an abrupt change in the current voltage characteristic of the plasma flow.

In Fig. 4 we consider the ion heating rates for the three temperature ratios and for the same parameters as in Fig 2. The reference classical heating rate is evaluated for $T_e/T_i = 0.1$. For the plasma parameters considered, the ions are being heated at about the same rate as the electrons are being heated classically. However, this rate is substantially larger than the classical heating rate for the ions. Thus we can conclude that the instability will be effective in heating ions. In Fig. 5 the electron heating rates are shown for the same parameters as in Fig. 2. In contrast to the ion heating rates, the electron heating rates substantially exceed the reference rate. This suggests that when this instability is excited in an MPD plasma, the electrons will get much hotter than the ions although the rise in the ion temperature will eventually cause the instability to damp as the dimensionless current drops. Hence, the instability will be self-healing although the final steady state will be very different from the initial state.

IV. Numerical Application to One-Dimensional MPD Flows

In the last two sections we showed how the modified two-stream instability can be excited in a plasma with typical MPD parameters, and we obtained the associated scattering rates. In this section we examine the instability for an equilibrium composed of a one-dimensional MPD flow.

The MPD flow is taken to be composed of a mixture of argon ions, electrons, and argon neutrals. The ions and neutrals are assumed to be highly coupled so that there is only one heavy species temperature. Ion viscosity is included as well as area variation. The fluid equations which describe the equilibrium are, first, mass conservation

$$\frac{\partial \rho U A}{\partial z} = 0 \tag{41}$$

where A is the cross-sectional area of the nozzle, and second, the conservation of momentum

$$\frac{\partial \rho U^2 A}{\partial z} + A\,\frac{\partial (p + B^2/2\mu_0)}{\partial z} = -\frac{12 U \mu}{H^2} A \tag{42}$$

where H is the transverse distance across the channel. This model for the viscous term arises from choosing a parabolic distribution for the velocity profile across the channel. This model represents fully developed flow in the channel and is chosen so that when we include viscosity in the simulations, the viscous effects will be maximized. The third equation is the electron continuity equation. We introduce the ionization fraction as $\alpha = n_e/(n_e + n_n)$. Then the electron density satisfies

$$\frac{\partial \rho \alpha U A}{\partial z} = A m_i \left(\dot{n}_e - \frac{12 D_a n_e}{H^2} \right) \tag{43}$$

The electron temperature satisfies

$$\frac{\partial \rho \alpha U T_e A}{\partial z} + \frac{2}{3} \rho \alpha T_e \frac{\partial U A}{\partial z} = A \frac{2}{3} m_i \left[\frac{J^2}{\delta} - E_l - E_i \dot{n}_e + \frac{\partial}{\partial z} \left(K_e \frac{\partial T_e}{\partial z} \right) \right] \tag{44}$$

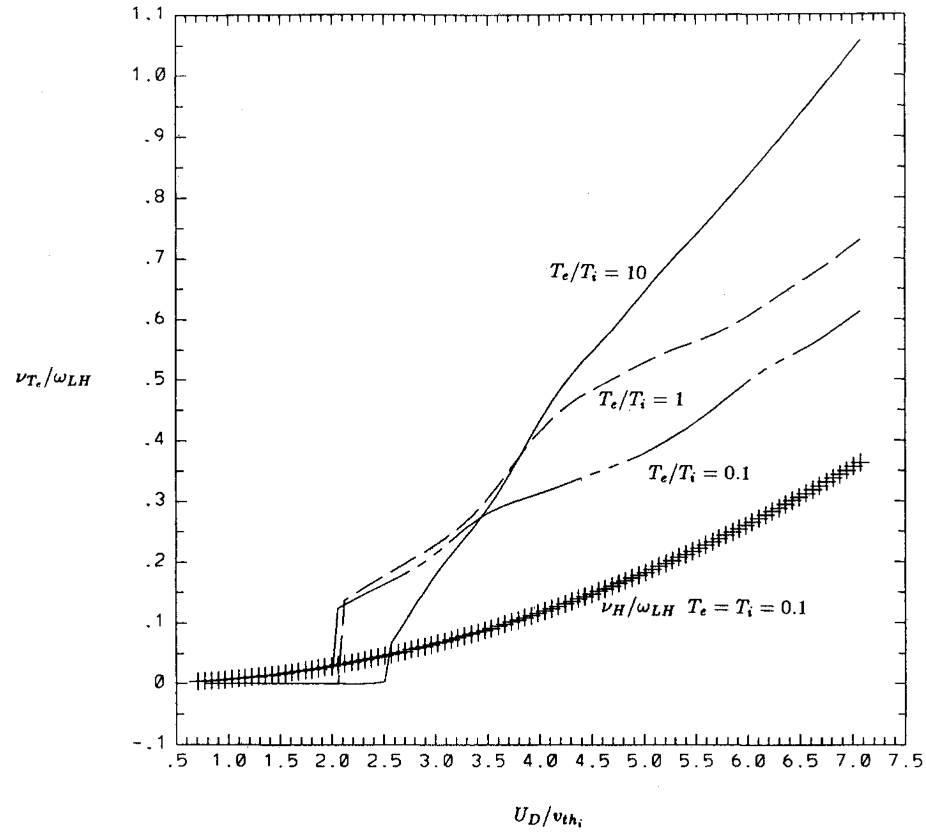

Fig. 5 The ν_{Te}/ω_{LH} and ν_H/ω_{LH} against U_D/v_{th_i} for $T_e/T_i = 0.1$, 1, 10.

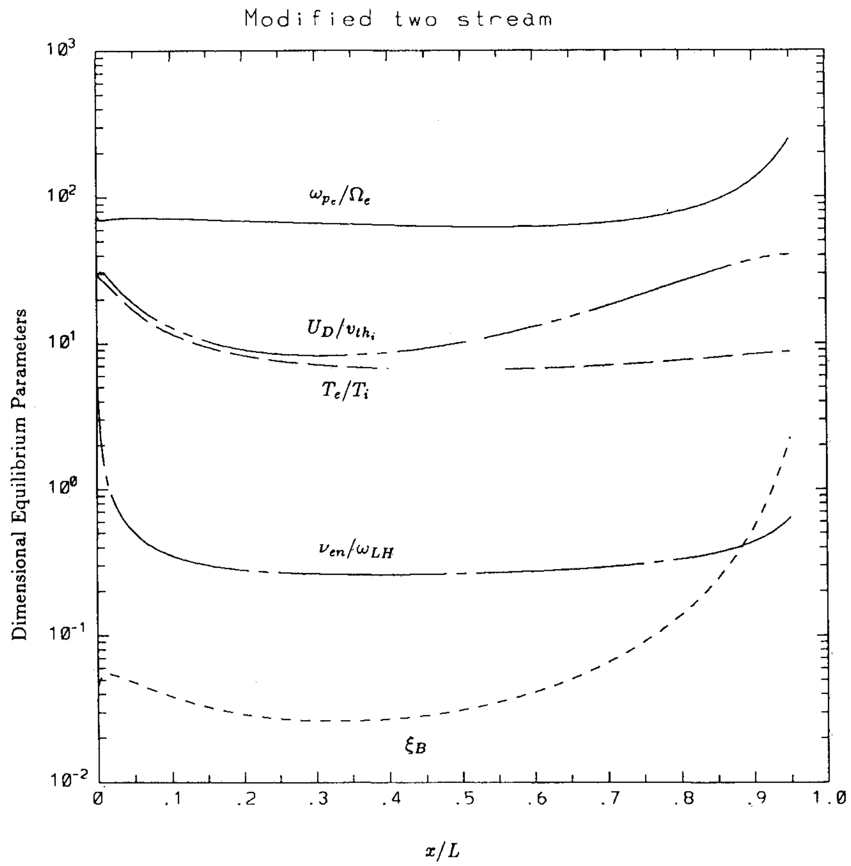

Fig. 6 The δ, U_D/v_{th_i}, T_e/T_i, ν_{en}/ω_{LH}, and ξ_B against z/L for the solution of the one-dimensional MPD equations with ambipolar diffusion, with no viscosity or electron heat conduction, and with a constant area.

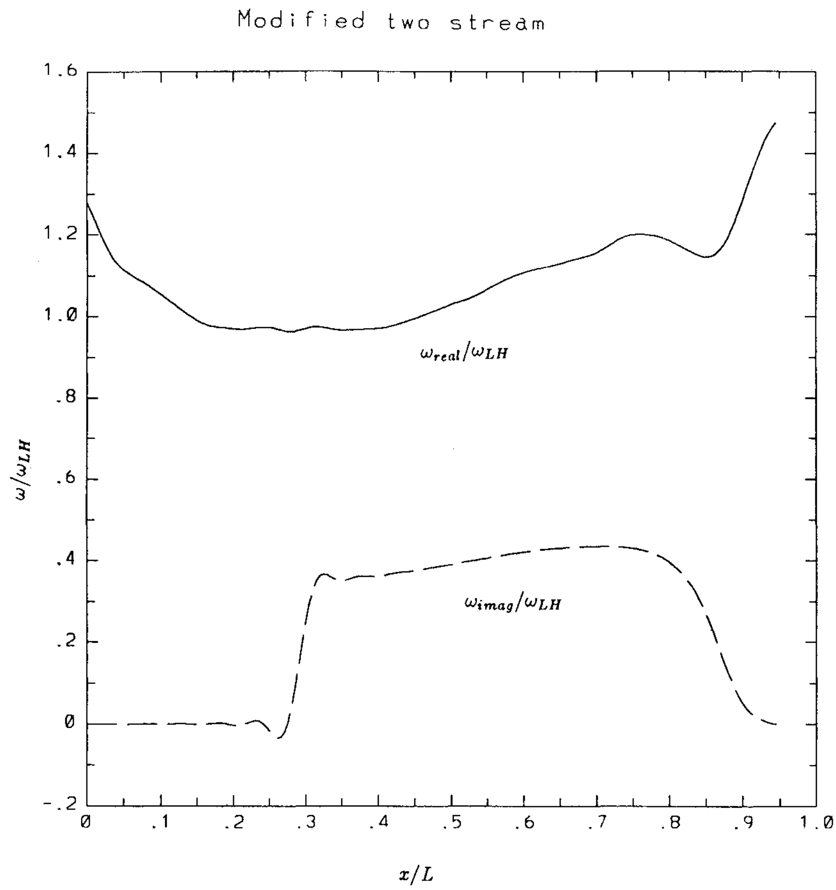

Fig. 7 The ω_{real} and ω_{imag} for the solution of the one-dimensional MPD equations with ambipolar diffusion, with no viscosity or electron heat conduction, and with a constant area.

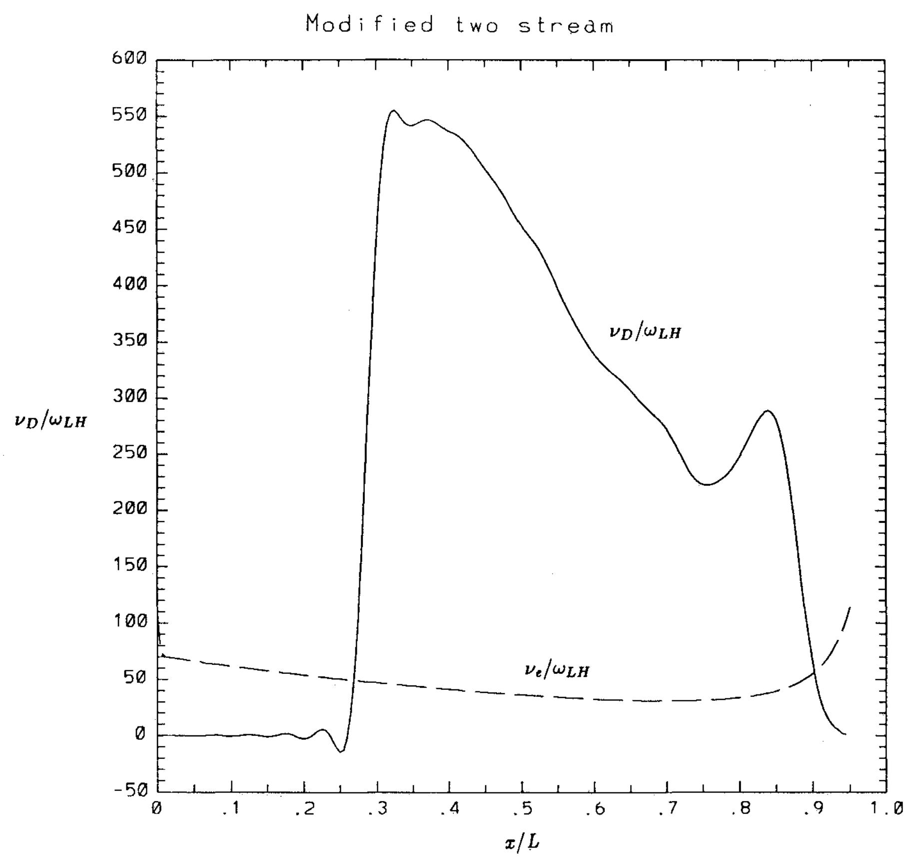

Fig. 8 The ν_D/ω_{LH} and ν_e/ω_{LH} for the solution of the one-dimensional MPD equations with ambipolar diffusion with no viscosity or electron heat condition, and with a constant area.

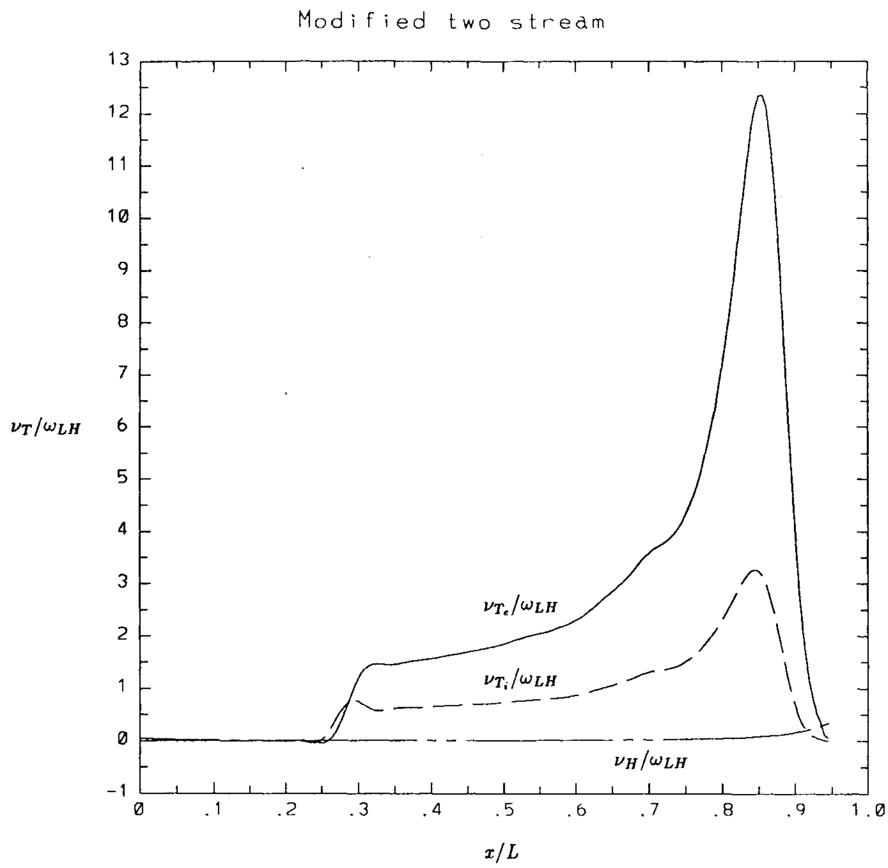

Fig. 9 The ν_{Te}/ω_{LH}, ν_{Ti}/ω_{LH}, and ν_H/ω_{LH} for the solution of the one-dimensional MPD equations with ambipolar diffusion with no vicosity or electron heat conduction, and with a constant area.

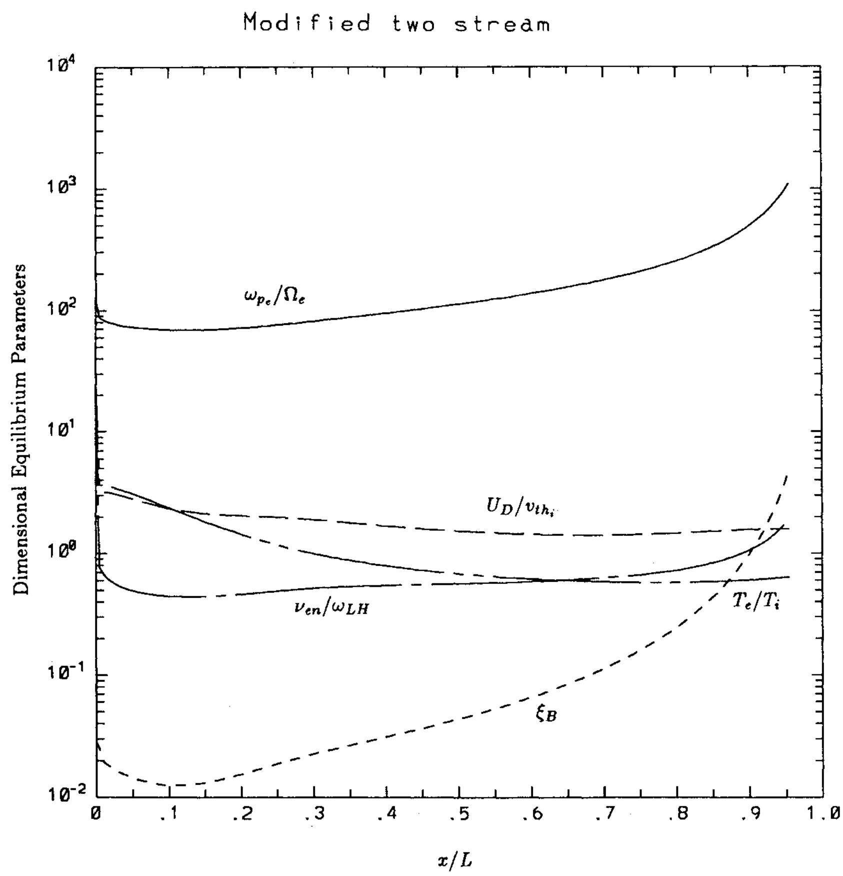

Fig. 10 The δ, U_D/v_{th_i}, T_e/T_i, ν_{en}/ω_{LH}, and ξ_B against z/L for the solution of the one-dimensional MPD equations with ambipolar diffusion, viscosity, and electron heat conduction and with a constant area.

The heavy species temperature is given by

$$\frac{\partial \rho U T_i A}{\partial z} + \frac{2}{3}\rho T_i \frac{\partial UA}{\partial z} = A\frac{2}{3}m_i\left(E_i + \frac{12U^2\mu}{H^2}\right) \tag{45}$$

In these energy equations, we have ignored the important effect of plasma radiation. This may be a significant energy loss for the plasma and might affect the instability by affecting the species temperatures. This effect will be included in future work. Finally, the magnetic field is determined from Ampere's law combined with Ohm's law to give

$$\frac{\partial UBA}{\partial z} + \frac{1}{\sigma\mu_0}\left(\frac{A}{\sigma}\frac{\partial \sigma}{\partial z} - \frac{\partial A}{\partial z}\right)\frac{\partial B}{\partial z} = \frac{A}{\sigma\mu_0}\frac{\partial^2 B}{\partial z^2} \tag{46}$$

The boundary conditions were taken to model a thruster which is 0.2 m long with an interelectrode separation of 0.02 m. The inlet magnetic field was chosen to be 0.1 T, which corresponds to a current of 80 kA. The magnetic field at the exit is set at zero. The inlet ionization was 0.01 and the mass flow rate per unit area at the inlet was $\dot{m}/A_{\text{inlet}} = 0.5$ kg/s/m^2. The inlet temperature of the heavy species is 0.027 eV (300 K).

In Fig. 6 we show the five nondimensional parameters that determine the modified two-stream instability obtained as the solution to the one-dimensional MPD equations above. For this simulation the ion viscosity and electron heat conduction were ignored although ambipolar diffusion was kept. The area of the channel was constant. In Fig. 7 the oscillation frequency and growth rate of the modified two-stream instability is shown for the parameters in Fig. 6 as a function of distance along the channel. Near the inlet the instability is damped by the dissipative effect of the electron neutral collisions. Near the exit the instability is also damped by the strong ∇B drifts. Over the central region of the thruster, the instability exists and has an average growth rate of one-third of the lower hybrid frequency. In this part of the thruster, the ion temperature is low since, with the ion viscosity ignored, there is no effective way to heat the heavy species and so the dimensionless current is large. In Fig. 8 we show the anomalous momentum exchange rate as compared to the classical electron-ion collision rate for this MPD simulation. Once the instability is excited, the anomalous rate exceeds the classical rate by an order of magnitude. This will lead to a drop in the conductivity and substantial rise in the voltage associated with passing this current. Finally, we show in Fig. 9 the electron and ion heating terms as compared to the classical rate. The electrons are being heated substantially especially near the end of the thruster. The ion heating will probably lead to the eventual damping of the instability.

In Fig. 10 we show the five nondimensional parameters that determine the modified two-stream instability obtained as the solution to the one-dimensional MPD equations above. For this simulation the ion viscosity, electron heat conduction, and ambipolar diffusion were kept. Otherwise the parameters were the same as in Fig. 6. The area of the channel was constant. For these parameters the dimensionless current density is sufficiently low that the instability can never be excited. This is because the ion temperature is high due to the substantial ion heating that occurs through viscous effects.

The first case examined is representative of a channel, such as a short channel with large interelectrode separation, in which the effects of viscosity would be negligible. In such a channel, there is insufficient ion heating to dampen the instability. The second case, a long thin channel in which fully developed flow is assumed throughout the channel, represents the opposite limit, in which viscosity has the maximum possible effect. In this case, viscosity raises the heavy species temperature enough so that the instability is not excited.

V. Conclusions

MPD plasma flows have a current flowing transverse both to the gas and to the self-consistent magnetic field. Current flows in any plasma constitute a source of free energy which can relax in plasma instabilities. The transverse current flow motivates us to examine the linear and nonlinear theory of the modified two-stream instability for an MPD plasma. The modified two-stream instability is a hydrodynamic instability, which is the analog of the two-stream instability for a magnetized plasma. The consequences of exciting this instability will be considerably enhanced plasma resistivity as well as ion and electron heating.

We show that for MPD flows where collisional effects and ∇B drifts may be important that typically the instability is excited when the dimensionless current density $J/(env_{\text{th}_i})$ exceeds 3. This instability is independent of the electron- to ion-temperature ratio. This is unlike the acoustic instabilities which were investigated by other authors. For a typical MPD simulation, the anomalous heating frequencies associated with this instability exceeded the classical rates by at least an order of magnitude. For the MPD channels that we examined numerically, we show that if there is a classical mechanism for substantially heating the ions, then the instability may not be excited. One such mechanism is the effect of viscosity, which would be most effective in long thin channels.

This instability may be observed experimentally in the following ways:

1) At a critical current level [current density $J/(env_{\text{th}_i})$ exceeds 3] plasma probes will be able to measure potential fluctuations at the lower hybrid frequency which will be propagating mainly along the magnetic field direction. A set of plasma probes would be able to extract the dispersion curve for the oscillations and compare it with the theoretical results. This has been done for acoustic oscillations.[7]

2) At a critical current level, there will be a marked increase in the plasma resistivity and associated ion heating. Whereas ion viscosity will also give rise to ion heating, the instability can be triggered and the effects observed in short fat channels where ion viscous effects would be expected to be absent.

Finally, we note that once the ions or electrons are heated sufficiently either by this instability or by other means, then the assumption that the plasma pressure is small as compared to the magnetic pressure may no longer be true. This will mean that the treatment of the waves as purely electrostatic will be questionable. In general, these finite plasma pressure effects will cause the modified two-stream instability to be damped.[10]

Acknowledgments

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